CSci 435: Formal Languages and Automata

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**Home Assignment 6: 90 points + 20 points (optional)**

**Due 11/7**

In any (N/D)PDA, assume that a start stack symbol z is already in the stack; so, you don’t have to insert z into the stack at the beginning of transition.

Q1.[30] Prove if the following languages are CFL or not.

If L is a CFL, give its CFG. Otherwise, prove it by Pumping Lemma.

If any closure property of CFL is applicable, apply them to simplify it before its proof.

1. [10] L = {*wwRw* | *w* ∈ {*a, b*}\*}

Proof by pumping lemma:

Assume given language is context free and there exist a pda for this.

Take a valid string from L

z = abbaab

divide the string z into 5 parts. u,v,w,x,y

u = a,v = b,w = ba, x = a,y = b

if for any value of i, uviwxiy belongs to L then L is context free otherwise not. Say i = 2 the uviwxiy becomes abbbaaab this is not belonging to L. this is contradiction to our given statement therefore this is not context free language

1. [10] L = { *anwwRbn* | *n* ≥ 0, *w* ∈ {*a, b*}\*}

Proof by pumping lemma:

Assume given language is context free and there exist a pda for this.

Take a valid string from L

z = aabaab

divide the string z into 5 parts. u,v,w,x,y

u = a,v = a,w = baa, x = b,y =∈

if for any value of i, uviwxiy belongs to L then L is context free otherwise not. Say i = 2 the uviwxiy becomes aaabaabb this is note belongs to L. this is contradiction to our given statement therefore this is not context free language.

1. [10] L = {*anbjajbn* | *n* ≥ 0, *j* ≥ 0}

This is context free language

Cfg is S-> aSa | aAa | A | ∈

A-> bAa|∈

1. [10, optional] L = { *an*| *n* is a prime number }

V ien : uviwxiy’ ∈ L

L = aabaab length = 6

|vwy|<= n

4 >= 6

|vx| => 1

3 => 1

L = {anbjajbn| n => 0 , j => 0 }

Q2. [20] Prove that the following languages are linear or not.

If L is linear, give the linear-CFG for L. Otherwise, prove it by Pumping Lemma for a Linear-CFL.

1. [10] L = { *w* | *na*(*w*) + *nb*(*w*) = *nc*(*w*) } is not linear.

Assume that language L as context free language.

Consider the string generated by language L.

W = aaa bbb ccccc

Where w ∈ L.

Now split w into 5 parts.

a a | a | b b | b | ccccc

here

p = aa , q = a , r = bb , s = b and t = ccccc

now check 3 cases of pumping lemma.

Case (1): | q s | => | |ab| > | 2> | True

Case (2): check pqi r sit eL for different values of i

i = 1 = ) aa (a)1 bb(b)1 ccccc = A3B3C6 ∈ L

i = 2 = ) ) aa (a)2 bb(b)2 ccccc = A4B4C6 ∈ L

Reason : Have number of a’s t number of b’s ≠ number of c’s

4 + 4 ≠ 6

8 ≠ 6

Case (3): fail

Since case(iii) fail our assumption that language l is context free language false

Not context free language

This is proved

1. [10] L = { *anbmcn* | *n, m* ≥ 0 } ∪ { *anbncm* | *n, m* ≥ 0 } is linear or not.

L = { *anbmcn* | *n, m* ≥ 0 }

Assume L is regular,

1. |y| > 0
2. |xy| <= p, and
3. Ai > 0, xyi z ∈ L.

S = 0p10p

xy0 z = xz = 0 (p-k) 10p

Our assumption that L is regular in wrong, L is not a regular language.

Q3. [30] Prove the following properties clearly.

1. [10] The family of CFLs is closed under reversal.

Let L be a CFL, with grammar G = (V,T,P,S).

Let LR be the reverse of L such that Grammar is GR = (V,T,PR,S).

1. [10] The family of DCFL is closed under regular difference:

i.e. for a DCFL L1 and a RL L2, L1 − L2 ∈ DCFL.

L1 = {abjck, i, j , k 3 0 and i1 j}. (a DCFL)

L2 = {abjck, i, j , k 3 0 and j1 k}. (a DCFL)

Lć= L1 È L2

= {abjck, i, j , k 3 0 and i1 j} of (j1 k))

Lć= o Lć

= {abjck, i, j , k 3 0 and i = j = k) = Ė

{w î {a,b,c}\* : the letters are out of order}

Lććć = Lććć Ç a\*b\*c.

= {anbncn, n30)

Lććć is not even CF , much less DCF.

L1 = {abjck, i, j , k 3 0 and I= j}

L2 = {abjck, i, j , k 3 0 and j= k}

Lć= L1 È L2

= L1 and L2 are determined context free

Diagram

Description automatically generated

1. [10] The family of CFLs is not closed under complement. Give an example for it.

L1 = {anbncm | n > = 0 and m > = 0 }

L2 = {ambncn | n > = 0 and m > = 0 }

L3 = L1 U L2 = {anbncN | n > = 0 }

Their intersection says both conditions need to be true but push down automata can compare only two. So it cannot be accepted by pushdown automata, therefore it is not context free.

1. [10] If L1 is linear and L2 is regular, L1⋅L2 is a linear language.

Given that,

If L1 is linear and if L2 is regular languages their concatenation L1.L2 will also be linear.

1. [10, optional] The family of DCFLs is **not** closed under reversal. Give an example.

L = 0 \* {a i b j c k | i < j} u 1 \* {a i b j c k | I < k}

This is DCFL

L R, the 0 and 1 won’t be seen until reaching the end of the string

L R is not a DCFL.

Then, the family of DCFL is closed under reversal.